

KEYS TO CALCULUS PRIMER PROBLEMS

1.1) $Y=8X$
 $dY/dX=8$

1.2) $Y=3X^{-1}$
 $dY/dX=-3X^{-2}=3/X^2$

1.3) $Y=100X-6X^2$
 $dY/dX=100-12X$

1.4) $Y=-100x + 12x^2 - .75x^3$
 $dY/dx=-100 + 24x - 2.25x^2$

1.5) $Y=5 \cdot X^{.5}$
 $dY/dX=2.5X^{-.5}= 2.5/\sqrt{X}$

1.6) $Y=1/x^{.33}$
 $dY/dX=1/3 \cdot (X^{-.33-1})= 1/3 \cdot (X^{-1.33})= 1/3X^{1.33}$

1.7) $Y=e^{-x}$
 $dY/dX= e^{-x}$

1.8) $Y=\ln X$
 $dY/dX=1/X$

1.9) $Y=8x \cdot 9x^2$
 $dY/dx=8x \cdot 18x + 8 \cdot 9x^2 = 144x^2 + 72x^2 = 216x^2$

1.10) $Y=4X/3X^2$
 $dY/dX=\frac{-4 \cdot 3X^2 - 4X \cdot 6X}{(3X^2)^2} = -12/9X^2 = -4/3X^2$

1.11) $Y=100x^3-6x^2$
 $dY/dX=300X^2-12X$

1.12) $Y=-10Z^2$ and $Z=7X$
 $dY/dX = dY/dZ \cdot dZ/dX = -20Z \cdot 7 = -140Z = -140 \cdot 7X = -980X$

$$2.1) \quad Y = X + X \cdot Z - Z^2$$

$$\partial Y / \partial X = 1 + Z$$

$$\partial Y / \partial Z = X - 2Z$$

$$2.2) \quad Y = 40 + 3X + 4Z^2$$

$$\partial Y / \partial X = 3$$

$$\partial Y / \partial Z = 8Z$$

$$2.3) \quad Y = 6X^3 + 4 \cdot X \cdot Z + Z^2$$

$$\partial Y / \partial X = 18X^2 + 4Z$$

$$\partial Y / \partial Z = 4X + 2Z$$

$$2.4) \quad Y = (4X + 10Z) \cdot (.5X + 2Z) \quad (\text{Use the product rule here!!})$$

$$\partial Y / \partial X = (4X + 10Z) \cdot .5 + 4 \cdot (.5X + 2Z) = 4X + 13Z$$

$$\partial Y / \partial Z = (4X + 10Z) \cdot 2 + 10 \cdot (.5X + 2Z) = 13X + 40Z$$

$$2.5) \quad Y = (4X + 10Z) / (.5X + 2Z) \quad (\text{Use the quotient rule here!!})$$

$$\partial Y / \partial X = \frac{4 \cdot (.5X + 2Z) - (4X + 10Z) \cdot .5}{(5X + 2Z)^2} = \frac{-3Z}{(5X + 2Z)^2}$$

$$\partial Y / \partial Z = \frac{10 \cdot (.5X + 2Z) - (4X + 10Z) \cdot 2}{(5X + 2Z)^2} = \frac{-3X}{(5X + 2Z)^2}$$

$$3.1) \quad Y = X + X \cdot Z - Z^2$$

$$dY = dX + Z dX + X dZ - 2 \cdot Z dZ$$

$$= (1 + Z) dX + (X - 2Z) dZ$$

$$3.2) \quad Y = 40 + 3X + 4XZ^2$$

$$dY = 3dX + 4Z^2 dX + 8XZ dZ$$

$$= (3 + 4Z^2) dX + 8XZ dZ$$

3.3) For 3.1), solve for the numerical value of dY if X is 10 and changes to 11 and Z is 4 and changes to 4.5.

$$dY = (1 + 4) \cdot 1 + (10 - 2 \cdot 4) \cdot .5 = 5 + (2 \cdot .5) = 6$$

$$4.1) \quad \text{Cost} = 8L^2 - 32L$$

$$\partial \text{Cost} / \partial L = 16L - 32 = 0$$

$$L^* = 2$$

$\partial^2 \text{Cost} / \partial L^2 = 16$ which is > 0 , so are at a minimum.

4.2) Profit = $-Q^3 + 57Q^2 - 315Q - 2000$ (solve for Q where $\partial \text{Profit} / \partial Q = 0$ and check the second order derivative for max or min).

$$\partial \text{Profit} / \partial Q = -3Q^2 + 114Q - 315 = 0$$

Hint on 4.2) use quadratic formula

For $ax^2 + bx + c = 0$

To find x values:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-114 \pm \sqrt{114^2 - 4(-3)(-213)}}{-6} = \frac{-114 \pm 96}{-6} = Q = 3 \text{ or } 35$$

$$\partial^2 \text{Profit} / \partial Q^2 = -6Q + 114$$

at 3 $\partial^2 \text{Profit} / \partial Q^2 = -6Q + 114 = 96 > 0$ so a minimum

at 35 $\partial^2 \text{Profit} / \partial Q^2 = -6Q + 114 = -96 < 0$ so a maximum

$$Q^* = 35$$

$$4.3) Y = 50X + 25Z + 5XZ$$

Subject to:

$$60 = X + Z$$

$$L = 50X + 25Z + 5XZ + \lambda(60 - X - Z)$$

First order conditions

$$\partial L / \partial X = 50 + 5Z - \lambda = 0$$

$$\partial L / \partial Z = 25 + 5X - \lambda = 0$$

$$\partial L / \partial \lambda = 60 - X - Z = 0$$

$$5 + Z = X$$

$$65 = 5 + Z + Z$$

$$60 = 2Z$$

$$Z = 30$$

$$X = 35$$